Schedule

Thursday April 4

1:00–2:00 Registration

2:00–2:10 Introductory remarks

2:10–3:00 Ethan Akin *Good Strategies for the Iterated Prisoner's Dilemma*

3:00–3:30 Coffee break

3:30–4:20 Jon Chaika *Some results that hold on every flat surface*

4:30–5:20 Marian Gidea *Perturbations of geodesic flows by recurrent dynamics*

Friday April 5

9:00–9:50 Tere M. Seara *Oscillatory motions in the circular restricted three body problem*

9:50–10:20 Coffee break

10:20–11:10 Jacques Féjoz *The problem of the stability of the Solar System – ideas of Lagrange and of today*

11:20–12:10 Rick Moeckel *Blowing-up collision singularities*

12:10–2:10 Lunch

2:10–3:00 Jean Pierre Marco *Examples of nearly integrable systems on $\mathbb{A}^3$ with asymptotically dense projected orbits*

3:15–4:15 **Department Colloquium** C. Eugene Wayne *Dynamical Systems and Metastable States*

4:15–4:45 Coffee break

4:45–5:15 Parallel Session I

- Colloquium room: Olena Karpel *Measures on Cantor sets and their classification*
- Room 1308: Ioannis Konstantoulas

5:25–6:15 Rafael de la Llave *Topological methods in the problem of instability*

7:00 Conference banquet at the Rotunda

Saturday April 6

9:00–9:50 Mark Demers *Perturbations of dispersing billiards via spectral methods*

9:50–10:20 Coffee break

10:20–11:10 Ke Zhang *Arnold diffusion via normally hyperbolic cylinders and Mather variational methods*

11:20–12:10 Konstantin Khanin

12:10–2:00 Lunch

2:00–2:50 Nandor Simanyi *Brief History of the Boltzmann-Sinai Hypothesis*

3:00–3:50 Anton Gorodetski *Sums of Cantor sets and convolutions of singular measures*

3:50–4:20 Coffee break

4:20–4:50 Parallel Session II

- Colloquium room: Anton Solomko *On spectral multiplicities for ergodic Abelian group actions*
- Room 1308: Alexey Korepanov

5:00–5:50 Anatole Katok *Arithmeticity and topology of smooth actions of higher rank abelian groups*

7:00 Party chez Vadim Kaloshin. Organizers will help in getting to Kaloshin's house by carpooling and public bus C8.

Sunday April 7

9:00–9:50 Jayadev Athreya *Counting special trajectories for right-angled billiards and pillowcase covers*

9:50–10:20 Coffee break

10:20–11:10 Peter Topalov *On the spectral rigidity of a class of integrable billiards*

11:20–12:10 Marco Martens *Probabilistic Universality*

Abstracts

Invited talks

Ethan Akin *Good Strategies for the Iterated Prisoner's Dilemma*
Using recent work of Press and Dyson I describe a class of strategies which strongly stabilize cooprative behavior in the Markov model for the repeated Prisoner's Dilemma.

Jayadev Athreya *Counting special trajectories for right-angled billiards and pillowcase covers*
In joint work with A. Eskin and A. Zorich, we compute (weak) quadratic asymptotics for counting special trajectories for billiards in polygons whose angles are integer multiples of 90 degrees. A key tool is computing volumes of moduli spaces of meromorphic quadratic differentials on CP, via enumerating pillowcase covers.

Jon Chaika *Some results that hold on every flat surface*
Recently Eskin-Mirzakhani-Mohammadi have proven a number of powerful results about the $SL_2(R)$ orbits of abelian differentials and the $SL_2(R)$ ergodic measures on the stratum. We discuss some results motivated and enabled by this work. One result is that for every abelian

differential there is a measure on the stratum, such that after rotating in almost every direction, the geodesic flow equidistributes for this measure on the stratum. Another result is that for any surface the conclusion of Oseledets multiplicative ergodic theorem holds for the Kontsevich-Zorich cocycle. This has an application, being explored by others, to the windtree model. This is joint work with Alex Eskin.

Rafael de la Llave *Topological methods in the problem of instability*

We will discuss how to use the method of correctly aligned windows to prove shadowing theorems that simplify and extend previous results. This is joint work with M. Gidea and T. M-Seara

Mark Demers *Perturbations of dispersing billiards via spectral methods*

We will discuss perturbations of the billiard map associated with a periodic Lorentz gas via the stability of the spectrum of the associated transfer operator. Recently, we constructed Banach spaces and norms on which the transfer operator for the unperturbed billiard enjoys a spectral gap. We will present a number of perturbations which fit into this functional analytic framework and for which the spectral gap persists, including: movements and deformations of scatterers, external forces with thermostatting, twists or kicks at reflections, and random perturbations composed of these various classes. This approach recovers many known results for these systems and establishes several new ones. This is joint work with Hongkun Zhang.

Marian Gidea *Perturbations of geodesic flows by recurrent dynamics*

We consider a geodesic flow on a compact manifold endowed with a Riemannian (or Finsler, or Lorenz) metric satisfying some generic, explicit conditions. We couple the geodesic flow with a time-dependent potential of a generic type, driven by an external flow on some other compact manifold. If the external flow satisfies some very general recurrence condition, we show that the coupled system has orbits whose energy grows unboundedly, at a linear rate with respect to time. This growth rate is optimal. We also show the existence of symbolic dynamics. The existence of orbits whose energy grows unboundedly in time is related to Arnold's diffusion problem. This is joint work with Rafael de la Llave.

Anton Gorodteski *Sums of Cantor sets and convolutions of singular measures*

Consider a one parameter family of dynamically defined Cantor sets and the measures of maximal entropy supported on these Cantor sets. We prove that under some natural technical conditions convolutions of these measures with another exact dimensional measure must be absolutely continuous for almost every value of the parameter. This is an essential generalization of the previous result by Peres and Solomyak for the families of affine Cantor sets. As an application, we show that the density of states measure of the Square Fibonacci Hamiltonian is a.c. for most small values of the couplings, therefore confirming the expectations of physicists. This is a joint work with D. Damanik and B. Solomyak.

Anatole Katok *Arithmeticity and topology of smooth actions of higher rank abelian groups*

The "non-uniform measure rigidity" program is an outgrowth of the measure rigidity program for hyperbolic algebraic actions of higher rank abelian groups that includes both, the famous Furstenberg "x2, x3 problem" and the joint work with M. Einsiedler and E. Lindenstrauss that has applications to the Littlewood conjecture on Diophantine approximations. While the latter program is still not completed (and probably hopeless without major new ideas) since all the results are based on a positive entropy assumption, the methods developed in hyperbolic measure

rigidity, together with the theory of non-uniform hyperbolicity (a. k. a. Pesin theory), have surprising applications way beyond the world of algebraic actions. In this talk I will discuss the most recent development in that direction.

Consider $k > 1$ commuting diffeomorphisms of a $(k+1)$- dimensional manifold M and assume that they preserve a measure that is sufficiently "rich" or "stochastic", for example if all non-zero elements of the suspension action have positive entropy with respect to the measure.

A striking topological conclusion is that the manifold is a "slightly modified" (finite factor of the) torus: there is homeomorphism from an open subset in finite cover of M that contains support of the lifted measure to the $(k+1)$-dimensional torus with a finite set removed. In particular, the fundamental group of a finite cover of M contains a free abelian subgroup or rank $(k+1)$.

On the ergodic side we obtain precise information about the structure of the action. It is fully "arithmetic", i.e. exactly the same as an action by commuting hyperbolic matrices with integer entries and determinant 1 or -1 on the torus or its finite factor. Moreover, the correspondence is smooth the sense of Whithey on a set whose complement has arbitrary small measure. This is a joint work with Federico Rodriguez Hertz. The main technical ingredient is our joint result with Boris Kalinin (Annals of Math, 2011)

If I have time, I will also discuss applications to entropy theory. A proper notion of entropy for actions of abelian groups that does not vanish automatically for smooth actions of Z^k or R^k, $k > 1$ was introduced by david Fried in 1983 and seems to be little known, probably because Fried did not extract any particularly interesting conclusions. We rediscovered this notion and it turns out that this Fried (or average) entropy satisfies remarkable rigidity property at least in the maximal rank case (Z^k actions on $(k+1)$- dimensional manifolds, $k > 1$) discussed above, Namely, its value is either zero or bounded from below by a positive constant $c(k)$ that exponentially growth to infinity with k.

Jean Pierre Marco *Examples of nearly integrable systems on $\mathbb{A}^3$ with asymptotically dense projected orbits*

Given an integer $\kappa \geq 2$, we introduce a class of nearly integrable systems on $\mathbb{A}^3$, of the form

$$H_n(\theta, r) = \tfrac{1}{2}\|r\|^2 + \tfrac{1}{n}U(\theta_2, \theta_3) + f_n(\theta, r)$$

where $U \in C^\kappa(\mathbb{T}^2, \mathbb{R})$ is a generic potential function and f_n a C^κ additional perturbation such that $\|f\|_{C^\kappa(\mathbb{A}^3)} \leq \frac{1}{n}$, so that H_n is a perturbation of the completely integrable system $h(r) = \frac{1}{2}\|r\|^2$.

This systems admits "asymptotically dense" orbits when $n \to \infty$, in the following sense. Let $\Pi : \mathbb{A}^3 \to \mathbb{R}^3$ be the canonical projection. Then, for each $\delta > 0$, there exists n_0 such that for $n \geq n_0$, the system H_n admits an orbit Γ_n at energy $\frac{1}{2}$ whose projection $\Pi(\Gamma_n)$ is δ–dense in $\Pi({H_n}^{-1}(\frac{1}{2}))$, in the sense that the δ–neighborhood of $\Pi(\Gamma_n)$ in $\mathbb{R}^3$ covers $\Pi({H_n}^{-1}(\frac{1}{2}))$.

Marco Martens *Probabilistic Universality*

Renormalization plays a crucial role in one-dimensional dynamics. In particular, at the period doubling transition to chaos one can understand the geometry in phase space and in parameter space in terms of renormalization. It turns out that this geometry is universal and the attractor is geometrically rigid. One dimensional dynamical systems are not realistic models of real world phenomena. However, transition to chaos often occurs in the real world along a period doubling cascade. Indeed, these transitions exhibit the same geometry as the one-dimensional period doubling transition to chaos. Renormalization will also play a crucial role in the study of

higher dimensional systems. For example, the transition to chaos of strongly dissipative Henon maps can be understood in terms of renormalization. Surprisingly, the universal geometry from one-dimensional dynamics is present, but in a much more subtle way. In higher dimensional dynamics, universality has a probabilistic nature.

Rick Moeckel *Blowing-up collision singularities*

An important feature of the Newtonian n-body problem is the existence of collisions. For example, in the four-body problem there can be quadruple collisions, triple collisions and binary collisions. Binary collisions can be regularized. More complicated types can be blown-up using methods developed by McGehee. I will describe some work on techniques for blowing up many singularities at once.

Tere M. Seara *Oscillatory motions in the circular restricted three body problem*

This talk presents a proof of the existence of oscillatory motions in the circular restricted three body problem. The main ideas of the proof were already in Moser's proof for the existence of such motions in the so-called Sitnikov problem, also used in our case, with highly restrictive hypothesis about the mass ratio, by J. Llibre and C. Simo.

Our proof follows the ideas of these papers, mainly the transversal intersection of the manifolds of infinity and the consequent existence of chaotic dynamics, but we are able to give the result for any value of the mass ratio. The reason is that we are able to deal with the exponentially small splitting of the manifolds of infinity without assuming any smallness condition about the mass ratio. This is a joint work with M. Guardia and P. Martin.

Nandor Simanyi *Brief History of the Boltzmann-Sinai Hypothesis*

The Boltzmann-Sinai Hypothesis dates back to 1963 as Sinai's modern formulation of Ludwig Boltzmann's statistical hypothesis in physics, actually as a conjecture: Every hard ball system on a flat torus is (completely hyperbolic and) ergodic (i. e. "chaotic", by using a nowadays fashionable, but a bit profane language) after fixing the values of the obviously invariant kinetic quantities. In the half century since its inception quite a few people have worked on this conjecture, made substantial steps in the proof, created useful concepts and technical tools, or proved the conjecture in some special cases, sometimes under natural assumptions. Quite recently I was able to complete this project by putting the last, missing piece of the puzzle to its place, getting the result in full generality. In the talk I plan to present the brief history of the proof by sketching the most important concepts and technical tools that the proof required.

Peter Topalov *On the spectral rigidity of a class of integrable billiards*

The Radon transform of an integrable billiard is an averaged quantity associated to the Liouville tori in the phase space of the billiard. I will discuss the relation of the Radon transform to some new iso-spectral invariants of the Laplace-Beltrami operator associated to smooth deformations of the Riemannian metric of the billiard table. This is a joint work with Georgi Popov.

Eugene Wayne *Dynamical Systems and Metastable States*

The study of stable, or stationary, states of a physical system is a well established field of applied mathematics. Less well known or understood are metastable states. Such states are not fixed points of the underlying equations of motion but are typically a family of states which emerge relatively quickly, dominate the evolution of the system for long times, and then ultimately give way to the asymptotic state of the system (from which they are typically distinct.)

Their presence is a signal that multiple time scales are important in the problem for instance, one associated with the emergence of the metastable state, one associated with the evolution along the family of such states, and one associated with the emergence of the asymptotic states. In this talk I will discuss recent research with Margaret Beck which proposes a dynamical systems understanding of metastable behavior in Burgers equations and the two-dimensional Navier-Stokes equation.

Ke Zhang *Arnold diffusion via normally hyperbolic cylinders and Mather variational methods*

I will discuss a recent proof of generic Arnold diffusion in two and half degrees of freedom (based on joint works with V. Kaloshin, and with V. Kaloshin and P. Bernard). I will describe how variational methods based on Mather theory and weak KAM theory allows to realize and generalize the geometric picture of normally hyperbolic cylinders, heteroclinic connections, and shadowing.

Parallel sessions

Olena Karpel *Measures on Cantor sets and their classification*

Two measures μ and ν on a Cantor set X are called *homeomorphic* if there exists a homeomorphism f of X such that $\mu = \nu \circ f$. We are interested in the problem of classification of Borel probability and infinite measures on a Cantor set with respect to a homeomorphism. For a wide class of probability measures which E. Akin called *good*, a criterion of being homeomorphic is known. A full non-atomic measure μ is good if whenever U, V are clopen sets with $\mu(U) < \mu(V)$, there exists a clopen subset $W \subset V$ such that $\mu(W) = \mu(U)$. For the class of good probability measures, the set $S(\mu)$ of values of measure μ on all clopen subsets of X is a complete invariant. We consider ergodic probability and infinite invariant measures for aperiodic substitution dynamical systems. S. Bezuglyi, J.Kwiatkowski, K.Medynets and B.Solomyak showed that these measures can be described as ergodic measures on non-simple stationary Bratteli diagrams invariant with respect to the cofinal (tail) equivalence relation. We also consider a wide class of infinite measures on a Cantor set and finite and infinite measures on a non-compact locally compact Cantor set. In every case, we find necessary and sufficient condition for good measures to be homeomorphic. It turns out, that for good infinite measures, the set $S(\mu)$ is not a complete invariant, we find a new invariant which is complete.

Anton Solomko *On spectral multiplicities for ergodic Abelian group actions*

The set of spectral multiplicities of a probability preserving Abelian group action is an important invariant of spectral (and hence metric) equivalence of dynamical systems. We consider the following spectral multiplicity problem: Which sets are realizable as sets of spectral multiplicities for ergodic (or weakly mixing) systems? This problem was studied mainly for Z-actions. We will present some new realizations for arbitrary discrete countable Abelian group actions as well as for flows.