2009 MARYLAND DYNAMICS WORKSHOP

All talks will be in room 3206 unless otherwise noted. Coffee breaks will be in the adjacent lounge, room 3201.

Friday, April 3

3:00-4:00 COLLOQUIUM. A. Katok *Invariant geometric structures for group actions: measure rigidity and Zimmer program.*

7:00–late ERGODIC PARTY CHEZ BOYLE.

Saturday, April 4.

9:00–9:20 COFFEE

9:20–9:30 WELCOMING REMARKS

9:30–10:20 D. Wang *Dynamics of Periodically Perturbed Homoclinic Solutions.*

10:20–10:50 COFFEE

10:50–11:40 K. Zhang *Estimating the speed of Arnold diffusion.*

12:00–3:00 LUNCH BREAK (Coffee will be served in the lounge at 2:30)

3:00–3:50 J. Xia *Homoclinic points for C^r generic surface diffeomorphisms.*

4:00–4:50 A. Bufetov *Finitely-additive measures on the asymptotic foliations of a Markov compactum.*

4:50–5:20 COFFEE

5:20–6:10 M. Guysinsky *Some results about Livšic Theorem for 2×2 matrix valued cocycles.*

Sunday, April 5.

9:00–9:30 COFFEE

9:30–10:20 I. Putnam *A homology theory for basic sets*

10:20–10:50 COFFEE

10:50–11:40 O. Sarig *A necessary & sufficient condition for spectral gap for Ruelle operators on countable Markov shifts (joint with V. Cyr)*

12:00–3:00 LUNCH BREAK (Coffee will be served in the lounge at 2:30)

3:00–3:25 PARALLEL SESSION-1.

Room 3206. V. Climenhaga *Black-box multifractal formalism*

Room 1311. H. Krueger *Schroedinger Operators defined by Interval Exchange Transformations.*

Room 1308. Z. Wang *A Quantitative Result on Abelian Group Action of Toral Automorphisms.*

3:35–4:00 Parallel session-2.

Room 3206. V. Cyr *Obstructions to Transient Markov Shifts*

Room 1311. J. Chaika *There exists a topologically strong mixing IET*

Room 1308. P. Sun *Intermediate entropy measures for skew products*

4:00-4:30 Coffee

4:30–5:20 J. Yorke *Period-doubling Cascades Galore*

5:30–5:55 Parallel session-3.

Room 3206. C. Gonzalez Tokman *Approximating invariant densities of metastable systems.*

Room 1311. S. Tikhomirov *Shadowing in smooth flows and structural stability.*

Room 1308. A. Egorov *Markov property of the geometrical coding of the geodesic flow on surfaces of constant negative curvature.*

6:00–8:00 Reception.

Monday, April 6.

9:00–9:50 J. Athreya *Lattice point counting and volume growth in Teichmuller space.*

10:00–10:50 C. Ulcigrai *Absence of mixing in locally Hamiltonian flows on surfaces*

10:50–11:15 Coffee

11:15–12:05 P. Bachurin *Limiting distributions for a particle in a tube*

12:15–1:05 E. Gutkin *Security for Riemannian surfaces*

afternoon Informal discussions or Cherry Blossom trip

Tuesday, April 7.

9:00–9:50 T. Fisher *Trivial centralizers for Axiom A diffeomorphisms.*

10:00-10:50 H. Hu *Decay of correlations for some nonuniformly expanding maps.*

10:50–11:15 Coffee

11:15-12:05 J. Buzzi *Maximal entropy measures for a class of robustly transitive diffeomorphisms.*

12:15–1:05 A. Avila *TBA*

2:30–6:30 Hike in Great Falls Area (weather permitting)

Virtual talks.

The following posters will be available at the conference webpage:
http://www.math.umd.edu/research/dynamics/conferences/md09/
(parenthesis show the dates the presenter attends the conference).

Sergiy Borodachov (3-5), Towson University, *On a certain separation condition for attractors of finite systems of contractive homeomorphisms.*

Nikolai Chernov (3-7), Alabama and **Dmitry Dolgopyat** (3-7), Maryland, *Anomalous current in periodic Lorentz gases with infinite horizon.*

Maria F. Correia (3-7), University of Evora, *Symbolic dynamics for iterated smooth functions.*

Cecilia Gonzalez Tokman (3-7), Maryland, *Scaling laws for bubbling bifurcations.*

Andrew Torok (3-6), (Houston) *Extreme value theory for non-uniformly expanding dynamical systems.*

Abstracts.

Jayadev Athreya *Lattice point counting and volume growth in Teichmuller space.*

In joint work with A. Bufetov, A. Eskin, and M. Mirzakhani, we extend the results of the Ph.D. thesis of G. Margulis to the context of Teichmuller space. In particular, we study the asymptoptics of the number of mapping class group orbit points in a ball of radius R in the Teichmuller metric.

Pavel Bachurin *Limiting distributions for a particle in a tube.*

We study the behavior of a billiard particle in a planar tubular domain, which has a peculiar property: it reverses the direction of the most of incoming particles. We relate this property to the existance of certain limiting distributions for the circle rotations, which are studied with the help of the geoesic flow on $ASL(2,\mathbb{Z})\backslash ASL(2,\mathbb{R})$. This is a joint work with K.Khanin, J.Marklof, A.Plakhov.

Alexander Bufetov *Finitely-additive measures on the asymptotic foliations of a Markov compactum.*

Consider a compact oriented surface of genus at least two endowed with a holomorphic one-form. The real and the imaginary parts of the one-form define two foliations on the surface, and each foliation defines an area-preserving translation flow. By a Theorem of H.Masur and W.Veech, for a generic surface these flows are ergodic. The talk will be devoted to the speed of convergence in the ergodic theorem for translation flows. The main result, which extends earlier work of A.Zorich and G.Forni, is a multiplicative asymptotic expansion for time averages of Lipschitz functions. The argument, close in spirit to that of G.Forni, proceeds by approximation of ergodic integrals by special holonomy-invariant Hoelder cocycles on trajectories of the flows.

Generically, the dimension of the space of holonomy-invariant Hoelder cocycles is equal to the genus of the surface, and the ergodic integral of a Lipschitz function can be approximated by such a cocycle up to terms growing slower than any power of the time.

The renormalization effectuated by the Teichmueller geodesic flow on the space of holonomy-invariant Hoelder cocycles allows one also to obtain limit theorems for translation flows: it is proved that along certain sequences of times ergodic integrals, normalized to have variance one, converge in distribution to a non-degenerate compactly supported measure.

The argument uses a symbolic representation of translation flows as suspension flows over Vershik's automorphisms, a construction similar to one proposed by S.Ito.

Jerome Buzzi *Maximal entropy measures for a class of robustly transitive diffeomorphisms.*

Bonatti and Viana (2000) introduced a class of non-uniformly hyperbolic robustly transitive diffeomorphisms. These diffeomorphisms have a dominated splitting but are not partially hyperbolic. In a joint work with T. Fisher, we analyze their measures of maximal entropy.

Todd Fisher *Trivial centralizers for Axiom A diffeomorphisms.*

We show there is a residual set of non-Anosov C^∞ Axiom A dieomorphisms with the no cycles property whose ele- ments have trivial centralizer. If M is a surface and r is at least 2, then we will show there exists an open and dense set of of C^r Axiom A dieomorphisms with the no cycles property whose elements have trivial centralizer. Additionally, we will examine centralizers for codimension-one hyperbolic attractors.

Eugene Gutkin *Security for Riemannian surfaces*

A manifold is secure if the geodesic segments between any endpoints in it can be blocked by a finite number of point obstacles. The conjecture is that a compact manifold is secure iff it is flat. In a joint work with Victor Bangert we establish this for surfaces of genus greater than or equal zero.

Misha Guysinsky *Some results about Livšic Theorem for* 2×2 *matrix valued cocycles.*

We will discuss some new results about Livšic Theorem for 2×2 matrix valued cocycles. The first result is negative, there exist two smooth $GL(2, \mathbb{R})$–cocycles over Anosov map whose values at periodic points are conjugate matrices, but there is no continuous transfer map between them. The second result is positive if the values at periodic points not only conjugate matrices, but their Lyapunov's exponents at all periodic points are pairwise close to some (λ_0, μ_0), then these cocycles are hyperbolic and hence cohomologous with Hölder–continuous transfer map.

Huyi Hu *Decay of correlations for some nonuniformly expanding maps.*

We use the results developed by Sarig and Guoezel for decay of correlations to some nonuniformly expanding maps that do not have a Markov partition. We prove that if Losota-Yorke inequality is satisfied

for the reduced system, then under some general conditions on the systems, decay of correlations are polynomial. The results can be applied to one and higher dimensional expanding maps. (This is a joint work with Sandro Vaienti.)

A. Katok *Invariant geometric structures for group actions: measure rigidity and Zimmer program.*

Zimmer program, very roughly, aims at showing that volume preserving smooth actions of sufficiently "large" and "rigid" groups, such as semisimple Lie groups of real rank greater than one, or lattices in such groups, are essentially algebraic, or, more precisely, are built from algebraic blocks. Until recently successes were limited to very low-dimensional situations, or to the local problems (perturbations of algebraic actions, or to the actions satisfying additional dynamical asymptions, such as presence of Anosov elements.

I will discuss the first case of partial realization of this program without such limitations.

Consider the standard action ρ_0 of $SL(n,\mathbb{Z})$ on the n-dimansional torus $\mathbb{T}^n = \mathbb{R}^n/\mathbb{Z}^n$ by automorphisms. Let ρ be another smooth volume preserving action of $SL(n,\mathbb{Z})$ which induces the same action on the the first homology group $\mathbb{Z}^n$ of the torus.

Theorem 1. (A.K. and Federico Rodriguez Hertz, 2009) Let $n \geq 3$. Then there exist an open ρ-invariant set $U \subset \mathbb{T}^n$ and an injective smooth non-singular map $h : U \to \mathbb{T}^n$ such that $h \circ \rho = \rho_0 \circ h$.

Thus the action ρ is algebraic on a "large" open set U: $h(U)$ is dense and its complement is at most countable and consists of periodic orbits of ρ_0.

The main tool in the proof of this theorem is an earlier result of independent interest about actions of higher rank *abelian* groups on the torus A maximal rank semi-simple (diagonalizable over $\mathbb{C}$) abelian subgroups of $SL(n,\mathbb{R})$ is called Cartan. The rank of such subgroup in $n-1$ and its elements have real eigenvalues. Let α_0 be the action of a Cartan subgroup on $\mathbb{T}^n$ by automorphisms and let as before α be a smooth action of $\mathbb{Z}^{n-1}$ (not assumed volume-preserving) whose elements (but not the action as a whole) are homotopic to corresponding elements of α_0.

Theorem 2. (A.K. and Boris Kalinin, 2007) Let $n \geq 3$. Then the action α preserves an absolutely continuous measure μ and there is a set S of full μ-measure and an injective map $h : S \to \mathbb{T}^n$ such that $h \circ \alpha = \alpha_0 \circ h$ and $h_*\mu$ is Lebesgue measure. Furthermore, the map h is differentiable in the sense of Whitney on a subset of measure arbitrary close to one.

Thus the action α is also "algebraic", albeit is in a weaker sense, on a set of positive Lebesgue measure.

Recently we extended results of the above kind beyond the maximal rank case.

Theorem 3. (A.K. and Federico Rodriguez Hertz, 2009) Let α_0 be a linear $\mathbb{Z}^k,\ k \geq 2$, action on $\mathbb{T}^n$ which has no multiple or proportional Lyapunov exponents. Then, if α is an action whose elements are homotopic to those of α_0, there is only one α-invariant measure μ projecting to Lebesgue under the semiconjugacy, and μ is absolutely continuous w.r.t. Lebesgue.

Enrique Pujals Hyberbolicity far from homoclinic bifurcations.

We show that any $f \in Diff^1(M^n)$ can be C^1-approximated by other diffeomorphisms exhibiting a homoclinic bifurcation (tangencies or heterodimensional cycles) or by one which is essentially hyperbolic (it has a finite number of hyperbolic attractors with open and dense basins of attraction).

This is a joint work with Sylvain Crovisier.

Ian Putnam *A homology theory for basic sets*

David Ruelle introduced Smale spaces as topological dynamical systems with canonical coordinates of contracting and expanding directions as an axiomatic description of the basic sets for Smale's Axiom A systems. The totally disconnected irreducible Smale spaces are shifts of finite type. In the 1970's, Krieger introduced an invariant for shifts of finite type which is a dimension group. The goal of the talk is to show that this invariant may be extended to a type of homology theory for all irreducible Smale spaces.

Omri Sarig *A necessary & sufficient condition for spectral gap for Ruelle operators on countable Markov shifts (joint with V. Cyr)*

Abstract: Suppose X is a countable Markov shift and $f : X \to \mathbb{R}$ is a weakly Holder continuous function. We give a necessary and sufficient condition on f for there to exist a "rich" Banach space on which the Ruelle operator of f acts with spectral gap. The condition can be shown to be open and dense for a variety of topologies?

The main application is to the theory of equilibrium states: Suppose X has finite topological entropy, then the collection of function f which possess a unique equilibrium measure with exponential decay of correlations, CLT, real analytic pressure function etc is open and dense in a variety of topologies.

Corinna Ulcigrai *Absence of mixing in locally Hamiltonian flows on surfaces*

We consider area-preserving flows on surfaces which are locally given by smooth Hamiltonians with only simple saddles. We prove that such flows are typically (in the measure theoretical sense) not mixing (even though weakly mixing). The result can be reformulated in terms of suspension flows over interval exchange transformations under a roof function with symmetric logarithmic singularities and it is related to the behaviour of deviations of Birkhoff averages.

Don Wang *Dynamics of Periodically Perturbed Homoclinic Solutions.*

We obtain a comprehensive description on the overall geometrical and dynamical structures of homoclinic tangles in periodically perturbed second order ordinary differential equations with dissipations. Let μ be the size of perturbation and Λ_μ be the *entire* homoclinic tangle. We prove in particular that

(i) for infinitely many disjoint open sets of μ, Λ_μ contain *nothing else* but a horseshoe of infinitely many branches;

(ii) for infinitely many disjoint open sets of μ, Λ_μ contain *nothing else* but one sink and one horseshoe of infinitely many branches; and

(iii) there are positive measure set of μ so that Λ_μ admits strange attractors with Sinai-Ruelle-Bowen measure. We also use the equation

$$\frac{d^2q}{dt^2} + (\lambda - \gamma q^2)\frac{dq}{dt} - q + q^2 = \mu q^2 \sin 2\pi\omega t$$

to illustrate how to apply our theory to the analysis and to the numerical simulations of a given equation.

James Yorke *Period-doubling Cascades Galore*

Many systems have period-doubling cascades. I will discuss why these exist in great profusion. This is joint work with Evelyn Sander of George Mason University.

Ke Zhang *Estimating the speed of Arnold diffusion*

For a close-to-integrable Hamiltonian system with more than 2 degree of freedom, the existence of orbits whose action varible makes $O(1)$ change is often referred to as Arnold diffusion. For quasi-convex analytic Hamiltonians that is $\epsilon-$close to integrable, Nekhoroshev theory predicts a stability time of $e^{C\epsilon^{-\alpha}}$, during which Arnold diffusion cannot happen. For $n-$degrees of freedom, the stability exponent $\alpha = \frac{1}{2n}$ in general, and $\alpha = \frac{1}{2(n-m)}$ if the orbit is close to m-resonances. For the generalized Arnold's example

$$H = \frac{1}{2}p^2 + \epsilon(\cos q - 1) + \frac{1}{2}I^2 + \epsilon\mu(1 - \cos q)f(\varphi),$$

where $p \in \mathbb{R}$, $I \in \mathbb{R}^d$, $q \in \mathbb{R}/2\pi\mathbb{Z}$, $\varphi \in \mathbb{R}^d/(2\pi\mathbb{Z})^d$, we construct an orbit where Arnold diffusion happens in $O\left(e^{C\epsilon^{-\frac{1}{2(d-1)}}}\right)$-time. The exponent $\frac{1}{2(d-1)}$ coincides with the upper bound predicted by Nekhoroshev theory.

Jon Chaika *There exists a topologically strong mixing IET*

This talk will present a topologically strong mixing IET and discuss some related results.

Vaughn Climenhaga (Penn State) *Black-box multifractal formalism*

The multifractal formalism characterises a dynamical system in terms of dimensional characteristics which may vary from point to point. Results on the multifractal spectra associated with certain systems have previously been obtained in a somewhat ad hoc manner using information about the thermodynamic properties of those systems, in particular the existence and uniqueness of equilibrium states corresponding to certain families of potentials. In this talk, we show that this procedure is in fact generally valid, and carry out the multifractal formalism as a "black box" which depends only on thermodynamic results, and not on properties which are specific to the system.

Van Cyr (Penn State) *Obstructions to Transient Markov Shifts* In this talk, we ask when the countable Markov shift associated to a directed graph $\mathcal{G}$ has a transient potential. We define a new (checkable) criterion on $\mathcal{G}$ which turns out to be necessary and sufficient for the existence of such a potential; most shifts satisfy it. In this case we show that, in a suitable topology, the phenomenon of transience is the main obstruction to strong positive recurrence (a condition which guarantees the existence of a (weak) equilibrium measure).

Arseny Egorov (Penn State) *Markov property of the geometrical coding of the geodesic flow on surfaces of constant negative curvature.*

We consider a surface which is a factor of the hyperbolic plane $\mathcal{H}$ over a discrete group of its orientation preserving isometries (i.e. a Fuchsian group) Γ. The surface can be lifted to the hyperbolic plane in numerous ways, in particular one can cosider a Dirichlet fundamental domain $\mathcal{D}$ of the group, which is a polygonal set. The images of $\mathcal{D}$ under the action of the group form a tesselation of the plane. The sides of the elements of the tesselation are labeled by generators of the group in a certain way. Almost every geodesic on the plane does not pass through the vertices of the tesellation. Assuming that the factor surface has a finite volume, every such geodesic crosses infinitely many sides of the tesselation. Thus the so called "cutting sequence" of the geodesic is

formed by listing the labels of the sides the geodesic crosses and introduced by Morse in his unpublished work "Symbolic dynamics", 1966. It can be easily seen that such sequences cannot be arbitrary sequences of the generators of Γ. There may or may not be a short condition, called Markov property, which determines validity of a given sequence. We prove that the existance of this condition depends on the shape of the fundamental domain. Namely, the Markov property is satisfied only for fundamental domains with all vertices at infinity. The first result on this was obtained by S.Katok and I.Ugarcovici, stating that the standard fundamental domain of the modular group does not satisfy a Markov property. It used a comparison of codings of both geometric and arithmetic nature. The general proof is completely geometrical.

Cecilia Gonzalez Tokman (Maryland) *Approximating invariant densities of metastable systems.*

We consider a piecewise smooth expanding map of the interval possessing two invariant sets of positive measure, and therefore two ergodic absolutely continuous invariant probability measures (acims). As this system is perturbed and the invariant sets merge, we describe how the unique acim of the perturbed map is best approximated by convex combinations of the two initial ergodic invariant measures.

Helge Krueger (Rice) *Schroedinger Operators defined by Interval Exchange Transformations.*

Kotani's Theorem implies that if $V(n)$ is the potential sequence of an erogidic Schroedinger operator H, then the absolutely continuous spectrum of H is empty, when $V(n)$ is non deterministic. Consider $V(n) = f(T^n\omega)$, where f is continuous and T an interval exchange transformation. Our results imply under non degeneracy assumptions, that $V(n)$ is non deterministic, and thus H has empty absolutely continuous spectrum.

Peng Sun (Penn State) *Intermediate entropy measures for skew products*

For a skew product diffeomorphism, if there is no zero Lyapunov exponent along fibers, then we are able to show that there are ergodic invariant measures of arbitrary intermediate entropy. This is a generalization of Katok's result for smooth systems with hyperbolic measures. We adapted some lemmas from our previous result of existence of zero entropy measure. To construct the required measures we will show that there is a cantor subset which is invariant under some return map

and this return map along fibers is conjugate to a Markov shift. Moreover the average return time can be estimated in order to calculate the entropy.

Sergei Tikhomirov (St. Petersburg) *Shadowing in smooth flows and structural stability.*

The shadowing problem is related to the following question: under which condition, for any pseudotrajectory (approximate trajectory) of a vector field there exists a close trajectory? We study C^1-interiors of sets of vector fields with various shadowing properties. In the case of discrete dynamical systems generated by diffeomorphisms, such interiors were proved to coincide with the set of structurally stable diffeomorphisms for most general shadowing properties.

We prove that the C^1-interior of the set of vector fields with oriented shadowing property contains not only structurally stable vector fields. Also, we have found additional assumptions under which the C^1-interiors of sets of vector fields with Lipschitz, Oriented and Orbit shadowig properties contain only structurally stable vector fields.

Some of these results are joint with my advisor S. Yu. Pilyugin.

Zhiren Wang (Princeton University) *A Quantitative Result on Abelian Group Action of Toral Automorphisms.*

Furstenberg established in 1967 the famous theorem that all closed subset in $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ invariant under both $\times 2$ and $\times 3$ actions has to be either the whole circle or finite. A higher dimensional analogue concerning the action of higher rank abelian subsemigroups of $M_d(\mathbb{Z})$ on $\mathbb{T}^d$ was proved by Berend in 1983. Recently, Bourgain, Lindenstrauus, Michel and Venkatesh gave a quantitative version of Furstenbergs theorem. We effectivize a special case of Berends theorem by combining methods from Bourgain-Lindenstrauss-Michel-Venkatesh paper and Berends qualitative proof. Assume G is an abelian subgroup of $SL_d(\mathbb{Z})$ satisfying:

(1) $\text{rank}(G) \geq 2$;
(2) G contains an element which acts totally irreducibly on $\mathbb{T}^d$;
(3) G is maximal: there exists no abelian subgroup $H < SL_d(\mathbb{Z})$ containing G such that $\text{rank}(H) > \text{rank}(G)$.

The study of the toral action of such a group G is important because it corresponds to the group of unity in a non-CM number field of degree d. Under these assumptions we are able to show:

Theorem 1. There exist constants c_1 and c_2 depending only on G such that: for all R large enough and $0 < \alpha < 1$ if $S \subset \mathbb{T}^d$ is

2^{-R}-separated and has cardinality at least $2^{\alpha dR}$ then the set

$$B_{c_1R}S = \{gx | g \in B_{c_1R}, x \in S\}$$

is $R^{-c_2\alpha}$- dense. Here B_{c_1R} denotes the ball in G of radius c_1R with respect to the word length given by a fixed generating set.

This gives two corollaries about the behavior of a single orbit Gx.

Corollary 2. There exist constants c_3 and c_4 such that if $x = \frac{v}{N} \in \mathbb{T}^d$ where $v \in \mathbb{Z}^d$ and $N \in \mathbb{N}$ are coprime, then $B_{c_3 \log N}x$ is $(\log\log\log N)^{-c_4}$-dense.

Corollary 3. There exist constants c_5 and c_6 such that if x is a Diophantine generic point in $\mathbb{T}^d$, i.e. $|x - \frac{v}{M}| \geq M^{-b}$ for all coprime pairs $v \in \mathbb{Z}^d$ and $M \in \mathbb{N}$ and some constant b, then $B_{c_5 \log N}x$ is $(\log\log\log N)^{-c_6}$-dense for all N larger then a constant N_0 depending on b and G.

Theorem 1 is proved by applying the following ergodic-theoretical theorem to the uniform probability measure on S and standard test functions supported on small balls in $\mathbb{T}^d$.

Theorem 4. There exists constants c_1, c_7, c_8, c_9 such that: Let μ be a Borel probability measure on $\mathbb{T}^d$. Assume for a given $\alpha \in (0,1)$ and some sufficiently large R_0, any partition P of $\mathbb{T}^d$ whose mesh is less than or equal to 2^{-R_0} has entropy $H_\mu(P) \geq \alpha d R_0$. Then for all $\delta \in [c_7 \log R_0, \frac{\alpha}{100}]$ there is a probability measure ν inside the convex hull of $\{g\mu | g \in B_{c_1R_0}\}$ such that

$$\forall f \in C^\infty(\mathbb{T}^d) \quad \int_{\mathbb{T}^d} f d\nu \geq \int_{\mathbb{T}^d} f dm - c_8 R^{-c_9\delta} ||f||_{H^{(d+1)/2}}$$

where dm denotes the Lebesgue measure on $\mathbb{T}^d$ and $||f||_{H^{(d+1)/2}}$ is the Sobolev norm

$$\left(\sum_{\mathbb{Z}^d} |\xi|^{d+1} |\hat{f}(\xi)|^2 \right)^{1/2}.$$

Theorem 4 is a quantitative version of the theorem of Einsindler and Lindenstrauss which asserts any ergodic G-invariant measure on $\mathbb{T}^d$ with positive entropy has to be either dm or supported on a finite set.