Resonances and formation of the gaps in the spectrum of quasi-periodic Schrödinger equation

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Abstract

This talk is a review of some results on spectrum and localized eigen functions of quasi-periodic Schrödinger equation , obtained in recent joint works with Wilhelm Schlag. We consider one-dimensional difference Schrödinger equation

$$\big[H(x,\omega)\varphi\big](n) \equiv -\varphi(n-1) - \varphi(n+1) + V(x+n\omega)\varphi(n) = E\varphi(n) ,$$

$n \in \mathbb{Z}$, $x,\omega \in [0,1]$ with real analytic function $V(x)$. It was established in [GolSch2] that the positivity of the Lyapunov exponent for E belonging to some interval (E', E'') implies the absolute continuity of the integrated density of states on this interval for a.a.ω. In this talk I will explain how we apply the technology and the results of [GolSch2], to describe the mechanism responsible for formation of a dense in (E', E'') set of intervals free from the points of $\bigcup_{x} \mathrm{sp}\, H(x,\omega)$ in the regime of positive Lyapunov exponent.The results on gaps are effective in finite volume approximation , and therefore are both constructive and quantitative.

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